# Border Games in Cellular Networks

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***Abstract*— In each country today, cellular networks operate on carefully separated frequency bands. This careful separation is imposed by the regulators of the given country to avoid the interference between these networks. But, the separation is only valid for the network operators within the borders of their country, hence the operators are left on their own to resolve *cross-border interference* of their cellular networks. In this paper, we focus on the scenario of two operators, who want to fine-tune the emitting power of the *pilot signals* (i.e., beacon signals) of their base stations. This operation is crucial, because the pilot signal power determines the number of users they can attract and hence the revenue they can obtain. In the case of no power costs, we show that operators should be strategic in their borders, meaning to fine-tune the emitting power of their pilot signals. In addition, we study Nash equilibrium conditions in an empirical model and show the efficiency of the Nash equilibria for different user densities. Finally, we modify our game model to take power costs into account. In the model with power costs, the players should still be strategic, but their strategic behavior results in a well-known Prisoner's Dilemma and hence in a sub-optimal Nash equilibrium.**

***Index Terms*— Wireless networks, shared spectrum, pilot power control, cooperation, game theory, Nash equilibrium**

## I. Introduction

Today's cellular networks operate on separate frequency bands to avoid interference between them. The operators of these networks obtain an exclusive right to use a given frequency band in their respective country. However, the division based on frequency bands does not apply across national borders. As far as the borders are concerned, the operators have to resolve their conflicts themselves. One of the issues is referred to as the problem of *accidental roaming* [14], [19]. Often, the operators make make mutual agreements to resolve these conflicts, but these agreements are difficult to enforce, because they require the mutual cooperation of the operators. The proper understanding of this special case is important for the future studies of the coexistence of cellular networks, because there exist many examples where cities reside close to a national border. One can find examples for the border scenario on each of the continents such as Geneva, Basel or Aachen in Europe; San Diego and Detroit in the USA; or Hongkong and Singapore in Asia.

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In this paper, we consider the problem of strategic behavior of operators on the border of their cellular networks. We consider the operators of 3G cellular networks, such as the *Universal Mobile Telecommunication System (UMTS)* for example, that are based on the *Code Division Multiple Access (CDMA)* technology [12], [26], [27]. Note however, that the problem we highlight in the paper applies to all CDMA networks. In these cellular networks, the base stations emit pilot signals to help users to assess the available channel quality and to attach to the base station with the best offered quality. According to the current definition of the standard, the pilot power for the base stations is determined at the network dimensioning phase and remains fixed afterward. However, as the number of users changes, the operators should adjust the network parameters. This *slow adaptation* of the pilot signal power is part of the network re-dimensioning process and hence it exists on a large time scale. On the other hand, the technology enables the base stations to quickly adapt their pilot signals to the actual usage. This *fast adaptation* technique is commonly referred to as *cell breathing* [12], [26], [27].

In this work, we assume that the operators want to apply power control of the pilot signal of their base stations to attract more users over time. Several methods have been proposed to implement fast adaptation (i.e., the cell-breathing method) in CDMA networks. We survey them in Section II. In our paper, however, we focus on the slow adaptation problem. We study how the network operators can determine their pilot power in the presence of other operators given a certain user distribution. We investigate, whether this strategic situation leads to a game and we study the properties of the equilibria of power control strategies.

The remainder of the paper is organized as follows. Section II surveys related work. In Section III, we present the system model and the corresponding game-theoretic concepts. In Section IV, we study whether the operators should be strategic or not if there is no power cost. In Section V, we propose a distributed convergence algorithm to achieve the identified Nash equilibria. In Section VI, we extend the power control game to include the notion of power cost. We conclude in Section VII.

## II. Related Work

Power control has been extensively studied in the context of cellular networking. Baccelli *et al.* [2] consider downlink power allocation and admission control in CDMA networks relying on stochastic geometry. Hanly and Tse [11] as well as Catrein *et al.* [3] consider power control and capacity in CDMA networks. There is a very little literature about pilot power optimization, though [17], [30].

Game theory is used to study the power control of user devices in wireless networks, notably in cellular systems as studied in [1], [10], [13], [16], [22], [23], [20], [31] and [33]. A general framework for resource allocation in wireless network is addressed in [4].

Recently, the coexistence of multiple Internet Service Providers (ISPs) was studied by Shakkottai and Srikant in [29]. They consider both transit and customer prices for the ISPs. They show that if the number of ISPs competing for the same customers is large, then it can lead to price wars. In another paper [28], Shakkotai *et al.* consider the problem of non-cooperative multi-homing in WLANs. Zemlianov and de Veciana study a scenario in [32], in which users are able to choose between a cellular network and a Wi-Fi network. They show that congestion sensitive strategies are better than proximity-based strategies. Félegyházi and Hubaux [6] consider the competition between different operators in terms of pilot power control of their base stations. They show that in the pilot power control game a socially desirable Nash equilibrium exists and that it can be enforced by punishments.

## III. Model

### A. System Model

We assume an area, where cellular network operators want to provide wireless access to certain users. We assume that there exists a national *border* that separates the cellular networks of these operators. In particular, we consider a scenario with two cellular network operators $A, B \in O$, where $O$ is the set of operators. The operators operate their network based on the principles of the CDMA method. We assume that the two operators acquired the same frequency band for their networks in their respective country. This means that their networks interfere along the border. We assume that each operator controls a set of *base stations (BS)* $\mathcal{B}_i$, where $i \in O$. We refer to the set of all base stations as $\mathcal{B} = \bigcup_i \mathcal{B}_i$. We also assume a set of *users* $U$ equipped with *wireless devices* to access the communication network. For the sake of convenience, we do not separate neither the operators from their networks, nor the users from their devices. Consequently, we assume that the base stations (i.e., the operators) as well as the devices (i.e., the users) are the decision makers. In order to get an insight, we study the case in which each operator has one BS and refer to the BS-s by the letters of their operators (i.e., base station $A$ and $B$). This single-cell model is often considered in the literature [15], [21]. The network scenario is shown in Figure 1.

We assume that the radios of the base stations and the mobile devices are compatible, meaning that any user is able to access the network via any of the base stations. We further assume that the antennas of the BS-s and wireless devices are omnidirectional. Please note that the results derived in this paper are still valid in the case when the operators are using sectorized antennas that point towards the national border. Intuitively, they are still competing for the users that are between their base station. Sectorized antennas have more impact in the general scenario, where the operators have several base stations each. The study of this general scenario is the main focus of our ongoing work.

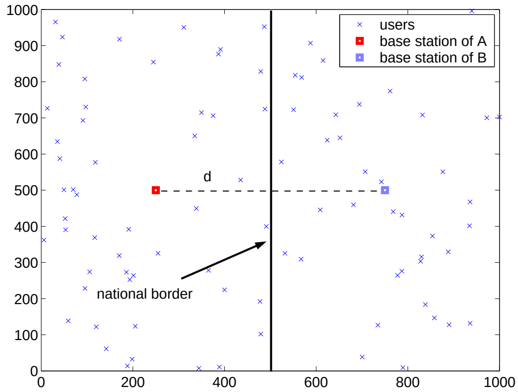



Fig. 1. Network scenario with two base stations.

Throughout this paper, we assume that the users are not associated with any of the operators (i.e., they are roaming users) and thus they attach to the base station with the best signal quality independently of the network.

In CDMA networks, power control is used to mitigate the near-far effect [26], to optimize the transmission power of the devices and to reduce interference. In this paper, we focus on the *downlink (or forward link) power control of the pilot signals* emitted by the base stations. The pilot signal helps the wireless devices to perform the following tasks:

- detection of the available base stations,
- synchronization with them and
- estimation of the channel quality and handover decision based on this estimation.

In particular, we focus on the problem of how the network operators can determine the pilot signal power that will potentially attract the highest number of users. We leave the study of the competitive fast adaptation problem as a future work.

As mentioned earlier, the pilot signal is used to attract users. If several users attach to a given base station, their transmissions are performed on different *channels*. In CDMA-based cellular networks, unlike GSM networks, channels are not separated in different frequencies, but using different codes. Hence each transmission uses the same frequency band. In theory, the codes from one base station are orthogonal, meaning that the transmissions to different receivers do not interfere with each other. In practice, there exists some interference between concurrent transmissions from a given base station because of multipath propagation. This interference is called the *own-cell interference*. In addition, there is an interference caused by the transmissions of other base stations, called the *other-cell interference*.

According to the *physical model* of signal propagation in a CDMA system [12], we can write the *signal-to-interference and noise ratio (SINR)* of the pilot signal of base station $i \in \mathcal{B}$

to user $u \in U$ as:

$$SINR_{iu}^{pilot} = \frac{G_p^{pilot} \cdot P_i \cdot d_{iu}^{-\alpha}}{N_0 + I_{own}^{pilot} + I_{other}^{pilot}} \quad (1)$$

where $G_p^{pilot}$ is the processing gain for the pilot signal, $P_i$ is the transmitted pilot signal of BS $i$, $d_{iu}$ is the distance between BS $i$ and user $u$, $\alpha$ is the path loss exponent, $N_0$ is the noise spectral density, and $I_{own}^{pilot}$ as well as $I_{other}^{pilot}$ are the own-cell and the other-cell interferences that affect the pilot signal of BS $i$.

Let us first express the own-cell interference $I_{own}^{pilot}$:

$$I_{own}^{pilot} = \gamma \cdot d_{iu}^{-\alpha}(\sum_{v \in U_i} T_{iv}) \quad (2)$$

where $\gamma$ is the *orthogonality factor* (also called the *own-cell interference factor*) that expresses the non-orthogonality between the different transmissions from BS $i$. Furthermore, $U_i$ is the set of users at BS $i$ and $T_{iv}$ is the traffic power assigned to user $v \in U_i$ by BS $i$.

Similarly, we can write the interference $I_{other}^{pilot}$:

$$I_{other}^{pilot} = \delta \sum_{j \neq i} d_{ju}^{-\alpha}(P_j + \sum_{v \in U_j} T_{jv}) \quad (3)$$

where $\delta$ is the *other-to-own-cell interference factor*, $d_{ju}$ is the distance between BS $j$ and user $u$. Furthermore $P_j$ is the pilot signal power of BS $j$, whereas $U_j$ is the set of users at BS $j$ and $T_{jv}$ is the traffic power assigned to user $v \in U_j$ by BS $j$.

Similarly to (1), we can express the SINR for the traffic signal $T_{iu}$:

$$SINR_{iu}^{tr} = \frac{G_p^{tr} \cdot T_{iu} \cdot d_{iu}^{-\alpha}}{N_0 + I_{own}^{tr} + I_{other}^{tr}} \quad (4)$$

Let us write the own-cell interference $I_{own}^{tr}$ for the traffic signal as:

$$I_{own}^{tr} = \gamma \cdot d_{iu}^{-\alpha}(P_i + \sum_{v \neq u, v \in U_i} T_{iv}) \quad (5)$$

and the interference from other BS-s $j$ as:

$$I_{other}^{tr} = I_{other}^{pilot} = \delta \sum_{j \neq i} d_{ju}^{-\alpha}(P_j + \sum_{v \in U_j} T_{jv}) \quad (6)$$

Furthermore, we can express the *carrier-to-interference ratio (CIR)* as a function of SINR:

$$CIR_{iu}^{pilot} = \frac{SINR_{iu}^{pilot}}{G_p^{pilot}} \quad (7)$$

where $G_p^{pilot}$ is the processing gain for the pilot signal from BS $i$ to user $u$. In UMTS systems, the processing gain for the pilot signal is $G_p^{pilot} = 256 \approx 14.3dB$.

Similarly, we can write the CIR of the traffic signal:

$$CIR_{iu}^{tr} = \frac{SINR_{iu}^{tr}}{G_p^{tr}} \quad (8)$$

where $G_p^{tr}$ is the processing gain for the traffic signal from BS $i$ to user $u$. Note that the value of $G_p^{tr}$ depends on the bitrate of the application running on the user device. In this paper, we refer to different types of communication as the

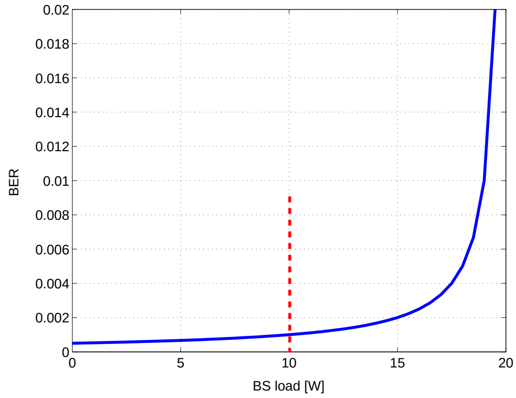



Fig. 2. Bit error rate (BER) as a function of the BS load in UMTS networks.

*traffic type*, namely audio (12.2 kbps), video (144 kbps) and data (384 kbps) flows. Accordingly, we distinguish different requirements for different traffic types as presented in [12]. We summarize these parameters in Table I.

TABLE I
UMTS PARAMETERS [12].

| traffic type | required SINR | processing gain | required CIR |
|---|---|---|---|
| pilot | ≈ -6 dB | 14.3 dB | -20 dB |
| audio, 12.2 kbps | 5 dB | 25 dB | -20 dB |
| video, 144 kbps | 1.5 dB | 14.3 dB | -12.8 dB |
| data, 384 kbps | 1 dB | 10 dB | -9 dB |

In wireless networks, the authorities impose a transmission power limit to the devices. In UMTS networks, the base stations must emit their signal below $43dBm = 20W$ [12]. This limit is called the *downlink power budget*. In addition, this power budget must be shared between the control channel signals, such as the pilot signal, and the traffic channel transmissions. The actual utilization of the power budget is called the *load* of the base station. As the load increases, the *bit-error-rate (BER)* at the user devices increases exponentially (see Figure 2). Hence, the BS load is typically kept such that the BER does not exceed a certain threshold, for example $10^{-3}$.

### B. Game-theoretic Concepts

We model competitive power control using game theory [5], [8], [9], [25]. We define a two-player non-cooperative *power control game* $G$ with the operators as *players*. In this game, the *strategies* of the operators determine the pilot transmission power of their base stations. Formally, we can write the strategy of operator $i \in O$ as the pilot signal power value of his BS:

$$s_i = P_i \quad (9)$$

where $0W < P_i < 10W$ is the pilot signal power of BS $i$. According to the UMTS standard, the BS-s transmit their pilot signal with approximately $33dBm = 2W$. We denote this standard pilot power by $P^s$. We call the set of strategies of all players a *strategy profile* $s = \{s_1, s_2\}^1$. In our game, the players have the same strategy set $S$.

The operators define their strategies in order to maximize their *expected payoff* $\pi_i$:

$$\pi_i = \sum_{u \in U_i} \theta_u \quad (10)$$

where $\theta_u$ is the *expected income* obtained by serving user $u$ of a certain traffic type. For the clarity of the presentation we calculate the expected income for a period of one month (i.e., the usual billing period) by assuming an average usage of the network. Suppose that each user has the same traffic type, for example audio. Then the expected payoff obtained at BS $i$ is:

$$\pi_i = |U_i| \cdot \theta_{audio} \quad (11)$$

We further assume that the income[2] per user increases according to the data rate of the given service, thus $\theta_{audio} < \theta_{video} < \theta_{data}$. In the paper, we express the utility of the players in *swiss francs (CHF)* to emphasize the monetary advantage.

In order to determine the average usage of the two networks, we developed a numerical simulator in MATLAB. We summarize the parameters of our simulation in Table II. In each simulation run, we distribute the users according to the pre-defined distribution and calculate the number of users that attach to each of the BS-s based on the CIR requirements presented in Table I. Note that we use a random uniform user distribution in our study, but our qualitative results hold for any user distribution. We repeat this experiment several times for each power setting and we obtain the average number of users at each BS. This defines an *experimental payoff matrix* for the two players. We apply the classic game-theoretic concepts on this payoff matrix.

TABLE II
SIMULATION PARAMETERS (BASED ON [12]).

| *Parameter* | *Value* |
|---|---|
| simulation area size | 1 km$^2$ |
| BS positions | (250m,500m) and (750m,500m) |
| default distance between BS-s, $d$ | 500m |
| user distribution | random uniform |
| number of simulations | 500 |
| default path loss exponent, $\alpha$ | 4 |
| BS max power | 43dBm = 20W |
| BS max load | 40dBm = 10W |
| BS standard power, $P^s$ | 33dBm = 2W |
| BS min power | 20dBm = 0.1 W |
| power control step size | 0.1W or 0.2W |
| orthogonality factor, $\gamma$ | 0.4 |
| other-to-own-cell interference factor, $\delta$ | 0.4 |
| user traffic types: | audio (12.2 kbps)<br>video (144 kbps)<br>data (384 kbps) |
| required CIR (audio, video, data): | -20 dB, -12.8dB, -9dB |
| expected incomes ($\theta_{audio}$, $\theta_{video}$, $\theta_{data}$): | 10, 20, 50 CHF/month |

[1]Note that one can easily extend the definitions in the power control game to several BS-s and operators.

[2]Note that the income is defined by the total amount of downloaded data, which can vary according to the length of communication sessions. If we change these income values, our results only change quantitatively, but not qualitatively.

We present our results using a symmetric scenario of the base stations and assuming that the users are uniformly distributed in the simulation area. Note that the result qualitatively hold for and base station placement and any user distribution. Naturally, in these cases, the Nash equilibrium strategies and utilities are going to by asymmetric.

In order to get an insight into the strategic behavior of the operators, we apply the following game-theoretic concepts. First, let us introduce the concept of *best response*. We can write $br_i(s_j)$, the best response of player $i$ to the opponent's strategy $s_j$ as follows.

*Definition 1:* The *best response* of player $i$ to the profile of strategies $s_j$ is a strategy $s_i$ such that:

$$br_i(s_j) = \arg\max_{s_i \in S} \pi_i(s_i, s_j) \quad (12)$$

One can see that if two strategies are mutual best responses to each other, then no player has a motivation to deviate from the given strategy profile. To identify such strategy profiles in general, Nash introduced the concept of Nash equilibrium in his seminal paper [24]. We can formally define the concept of *Nash equilibrium (NE)* as follows.

*Definition 2:* The pure-strategy profile $s^*$ constitutes a Nash equilibrium if, for each player $i$,

$$\pi_i(s_i^*, s_j^*) \geq \pi_i(s_i, s_j^*), \forall s_i \in S \quad (13)$$

where $s_i^*$ and $s_j^*$ are the Nash equilibrium strategies of player $i$ and $j$, respectively. In other words: In a Nash equilibrium, none of the players can unilaterally change his strategy to increase his utility.

We use the concept of *Pareto-optimality* to characterize the efficiency of different strategy profiles.

*Definition 3:* The strategy profile $s$ is *Pareto-optimal* if there does not exist another strategy profile $s'$ such that for each player $i \in O$:

$$\pi_i(s_i', s_j') \geq \pi_i(s_i, s_j) \quad (14)$$

with strict inequality for at least one player.

In other words, one cannot increase the utility of a player without hurting the other player.

## IV. IS THERE A POWER CONTROL GAME?

In this section, we study if operators should be strategic on the border or not. We first assume that one of the operators does not play and show that the other operator has an incentive to be strategic. Second, we consider the case in which both operators have the possibility to adjust their pilot power and show that they are better off by doing so[3].

### A. Only player $A$ is strategic

First, we consider the case where only operator $A$ is strategic and adjusts the pilot power of his BS to attract more users, whereas operator $B$ operates his BS according to the standard pilot power of $P^s = 2W$. To quantify the advantage of the strategic player, we define the concept of *strategic gain* $g_i$.

[3]Due to symmetry, we only show the results for player $A$.

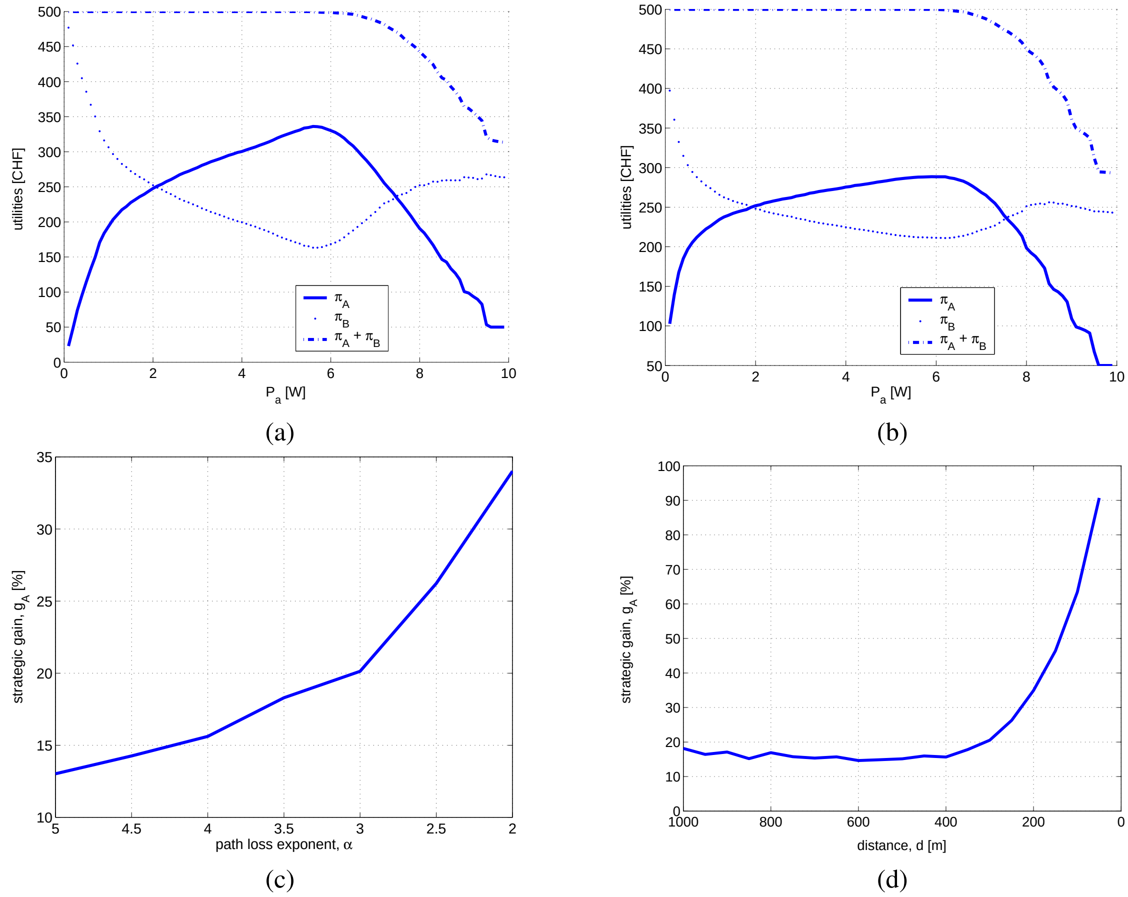



Fig. 3. Utilities of the players as a function of the pilot power of player $A$: (a) for $\alpha = 2$ and (b) for $\alpha = 4$. We also show the strategic gain $g_A$ as a function of (c) the path loss exponent $\alpha$ and (d) the distance $d$ between the two BS-s.

*Definition 4:* The *strategic gain* $g_i$ is the normalized difference between the maximum utility of player $i$ and his utility using the standard power $P^s$ assuming that the other player $j$ uses $P^s$.

$$g_i = \frac{\max_{s_i}\left(\pi_i(s_i, P^s)\right) - \pi_i(P^s, P^s)}{\pi_i(P^s, P^s)} \tag{15}$$

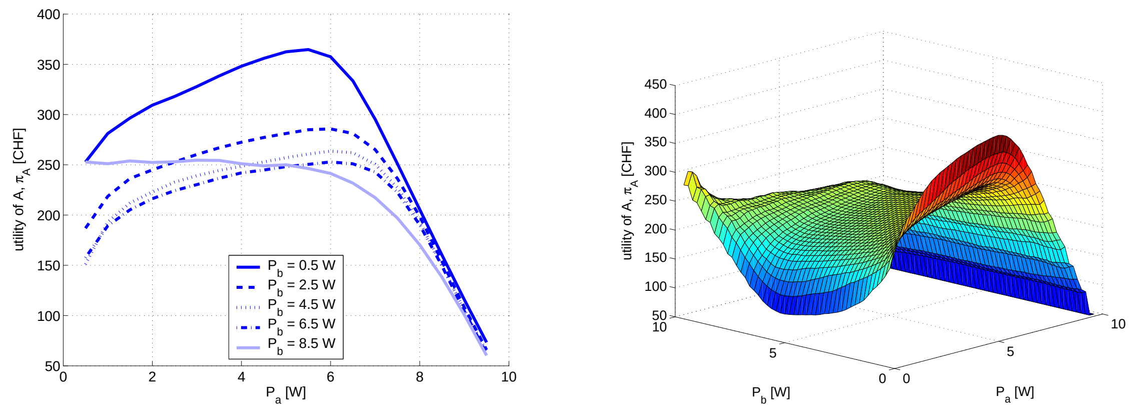
Suppose that there are on the average 10 users of the data traffic type in the simulation area. We show the utilities of players $A$ and $B$ as a function of the pilot signal power $P_a$ as well as the sum of their utilities in Figure 3. Figure 3a shows these utilities for $\alpha = 2$, whereas Figure 3b presents the same results for $\alpha = 4$. We observe that in both cases the operators are able to serve all users in the area using certain power values. Furthermore, the utility function of operator $A$ has a unique maximum point. It is interesting to observe that the maximum utility point requires a higher pilot power than $P^s = 2W$. Because the two operators serve all the users in this case, the strategic gain $g_A$ of player $A$ means the decrease of $g_A$ in the utility of the non-strategic player $B$. Hence, we conclude that operator $A$ should be strategic and adjust his pilot signal. Note that we get qualitatively the same result for different user traffic types.

Figures 3a and Figure 3b show that the value of the strategic gain $g_A$ depends on the parameter $\alpha$. We show this dependency in Figure 3c. One can observe that $g_A$ increase as $\alpha$ decreases. The reason is that by low $\alpha$ values the pilot signals propagate easier giving a higher gain to $A$ if he uses higher pilot power. The value of $g_A$ also depends on the distance $d$ between the two BS-s as shown in Figure 3d. As the distance decreases, $g_A$ increases exponentially. The reason for this increase is the same as discussed before. In the remainder of the paper, we choose the conservative default values $\alpha = 4$ and $d = 500m$ for the simulations. We will show that even with these conservative values, the players have a motivation to fine-tune their pilot powers.

## *B. Both operators are strategic*

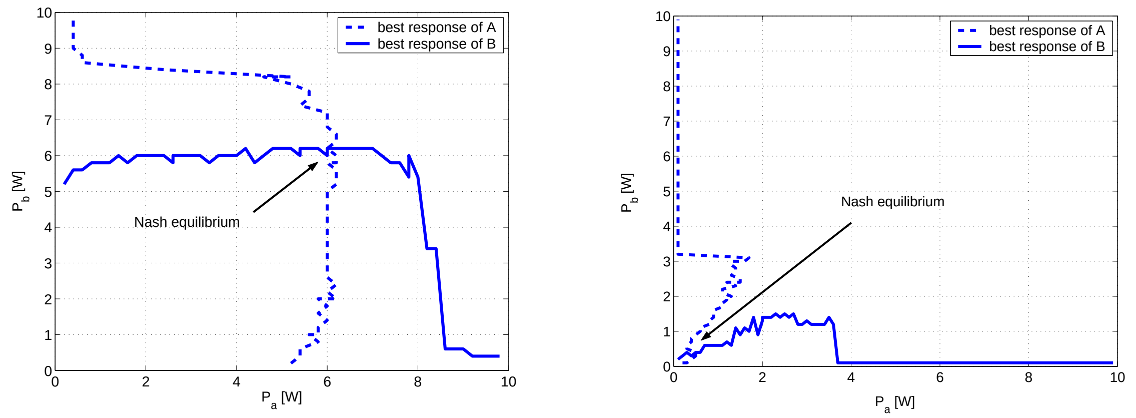
In the second set of experiments, we assume that both operators can adjust their pilot power. We still consider 10 data users in the simulation area. We provide the utility of player $A$ as a function of his pilot power $P_a$ in Figure 4a. We obtain different utility curves as the pilot power of the other BS $P_b$ increases. We can observe that each of the utility functions has a unique maximum point for $P_a$. Moreover, this maximum point depends on the pilot power of the other BS, $P_b$. For low values of $P_b$, the maximum utility value decreases as $P_b$ increases. In Figure 4b, we show the utility surface for operator $B$ as a function of the pilot power values of the two BS-s. Due to symmetry, the utility surface for operator $B$ is the mirroring of Figure 4b to the $x = y$ plane.

Using the two utility surfaces, we derive the best response functions (i.e., the set of maximum utility points) for the




(a) (b)

Fig. 4. Utility of player $A$ as a function of his pilot power. Both operators are strategic, hence we present this utility for various values of $P_b$ in (a). We show the complete utility surface in (b).




(a) (b)

Fig. 5. Best response functions for the two players with (a) 10 data users, (b) 100 data users.

operators as shown in Figure 5 for various user densities. Based on the concept of best responses introduced in Section III-B, we can identify the Nash equilibria in the power control game as shown in Figures 5a for 10 data users and Figures 5b for 100 data users. We see that there exists a unique Nash equilibrium point for any user density defined as the crossing point of the two best response functions. Note that for 10 data users the Nash equilibrium strategy profile defines $P_a = P_b = 6W$, which are higher than the standard pilot powers. For 100 data users the Nash equilibrium strategy profile defines $P_a = P_b = 0.5W$. The reason is that the BS-s can serve enough users by using a relatively small power and hence there is no motivation for them to go above these pilot power values.

Next, we study the pilot power values in the Nash equilibrium as a function of the number of users. We show the results in Figure 6. Due to symmetry in the user distributions, the Nash equilibrium pilot power is the same for both players. We observe that the Nash equilibrium pilot powers decreases as the number of users increases. For high user densities, the Nash equilibrium pilot powers stabilize at the value of $0.5W$.

In the following set of experiments, we study the *efficiency* of Nash equilibria and the standard power setting. To this end,

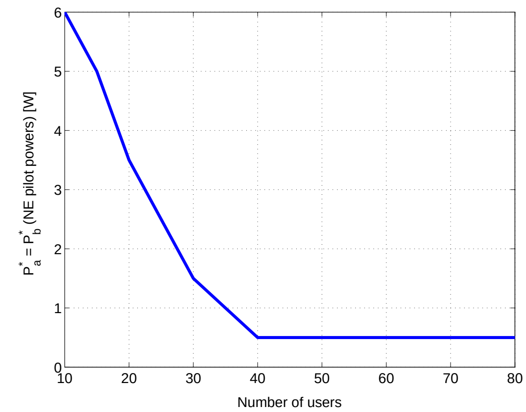



Fig. 6. Nash equilibrium pilot power values as a function of the user density.

we investigate the *utility region*, i.e. the utility values for various pilot power levels. We identify the utilities corresponding to the Nash equilibrium, the standard pilot power setting using $P^s$ and the utilities that correspond to Pareto-optimal strategy profiles. In particular, we can define the *Pareto boundary* as the set of Pareto-optimal utility points. In our case, the Pareto-optimal utility points characterize the system-efficient

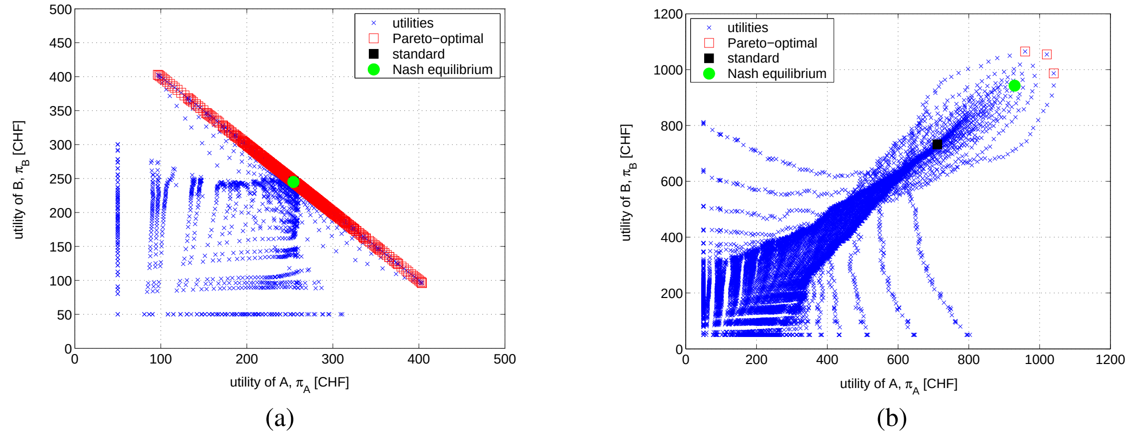



Fig. 7. The utility region with all possible utilities for (a) 10 data users and (b) 100 data users. We highlight the Nash equilibrium, the utility of the standard powers and all Pareto-optimal points.

solutions.

Figure 7a shows the achieved utilities as a function of the pilot power values $P_a$ and $P_b$ for 10 data users. We observe that in this case the Pareto boundary defines a straight line, because in a Pareto-optimal strategy profile each user in the system is attached to one of the BS-s. Furthermore, the standard pilot powers and the Nash equilibrium strategy profile result in the same utilities for the players and in addition they both lie on the Pareto boundary. This means that the players achieve a desirable state from the system point of view. Recall, however, that in this case the Nash equilibrium strategy profile requires higher pilot powers than the standard setting.

We present the utilities for 100 data users in Figure 7b. In this case the Pareto-optimal points do not form a straight line anymore, because some users cannot be served. Another observation is that the Nash equilibrium is still close to Pareto-optimality, but the standard solution becomes very inefficient.

Following the previous experiment, we formally express the efficiency of the standard and the Nash equilibrium solutions compared to the best Pareto-optimal point. To this end, let us define the following two concepts:

*Definition 5:* The *price of anarchy* [18] is the ratio between the total utility achieved by the two players in the best Pareto-optimal point and the Nash equilibrium.

*Definition 6:* The *price of conformance* is the ratio between the total utility achieved by the two players in the best Pareto-optimal point and using the standard pilot powers $P^s$ (i.e., being non-strategic).

We perform a set of experiments to measure these values for increasing user densities. Figure 8 presents the price of anarchy and the price of conformance as a function of the user density assuming they have data traffic. We see that both prices increase as the number of users increases. As we have seen in Figure 7a, both the standard utility point and the Nash equilibrium achieves Pareto-optimality if there is a small number of users. Hence, the two prices are very close to one. As the user density increases, we observe that both prices increase and then stabilize around a constant value. Note, however, that the price of anarchy stabilizes close to one, whereas the price of conformance stabilizes around 1.4.

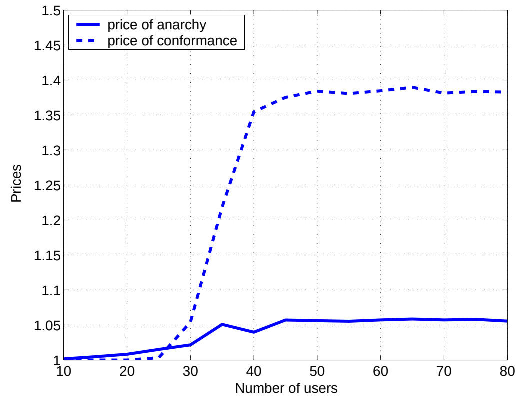



Fig. 8. The price of anarchy and the price of conformance as a function of the user density.

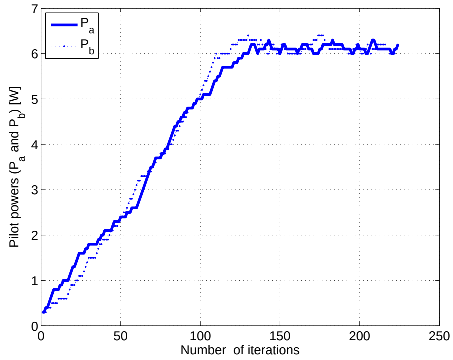
This shows that for a high number of users, the players can achieve a higher utility if both of them are strategic.

## V. Convergence to a Nash Equilibrium

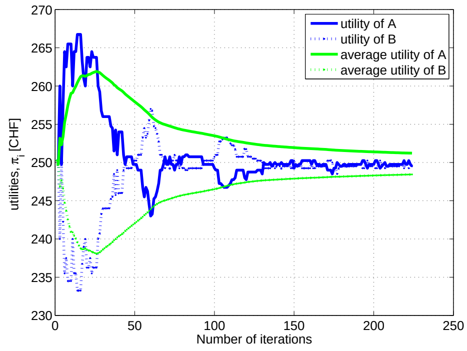
We have seen in the previous section that the expected utility function for a certain player is continuous and has a unique maximum point. In this section, we propose a distributed algorithm to achieve the Nash equilibrium in a given scenario. The algorithm is similar to the better-response dynamics [7], i.e., where each player tries to improve his utility in each step. We provide the pseudo-code as shown in Algorithm 1.

Figure 9a shows the evolution of the pilot power values applying Algorithm 1. We observe that the pilot power values follow the linear increase defined in the algorithm. After reaching the Nash equilibrium pilot power values, the algorithm stabilizes after certain steps.

Figure 9b shows the evolution of the utilities during the convergence process. We see that the algorithm deviates from the Nash equilibrium utilities while the pilot powers increase. As soon as the pilot powers reach the Nash equilibrium strategies, the utilities remain close to the Nash equilibrium utilities as well.




(a)




(b)

Fig. 9. Convergence to the Nash equilibrium using Algorithm 1. We present (a) the evolution of the pilot power values and (b) evolution of the utilities.

**Algorithm 1** Distributed convergence to the NE

**for all** player $i$ **do**
  set pilot power $P_i = 0.1W$
  set the direction of optimization $dir_i = +1$
**end for**
set power control step size $\Delta P = 0.1W$
**while** () **do**
  **for** all player $i$ **do**
    update $P_i$ with a probability $0 < q < 1$
    $P_i = P_i + dir_i \cdot \Delta P$
    **if** $u_i$ decreased **then**
      {the optimization passed the maximum utility value}
      $dir_i = -dir_i$
    **end if**
  **end for**
**end while**

## VI. Power Control Game with Power Cost

We have seen that the operators are able to serve all users in the area if the user density is low. We observe, however, that the Nash equilibrium pilot powers are higher than the standard value. Recall that the utility function defined in (10) does not include the possible cost due to the operation with high pilot power. Let us now extend the expected payoff function defined in (10) to capture this important aspect of the power control game. We introduce two cost values for each player. The first cost denoted by $c_i^{op}$ shows the *operating cost* of a BS $i$. This includes the aging of devices and hence the maintenance costs. The other cost, $c_i^{subj}$, expresses the *subjective cost* of player $i$. This covers every other aspect such as the risk of lawsuits or potential bad reputation due to high emission power. Without loss of generality, we assume that these cost are an increasing function of the downlink transmission power of the base stations.

According to the above description, we can extend the notion of expected payoff as:

$$\hat{\pi}_i = (\sum_{u \in U_i} \theta_u) - c_i^{op} - c_i^{subj} \tag{16}$$

We define a non-cooperative power control game with the new expected payoff function introduced in (16) and denote it by $\hat{G}$. We assume that the players can calculate the Nash equilibrium of the original game $G$ with no power cost. Hence we define the strategy in the extended game $\hat{G}$ as the choice between the standard and the Nash equilibrium strategies. Formally, we can write the strategies in $\hat{G}$ as:

$$s_i = \{P_i^*, P^s\} \tag{17}$$

Let us further assume that the players obtain an expected payoff $\Pi$ by serving half of the total number of users. As we have seen in Section IV-B, if they play the Nash equilibrium strategy profile by low user densities, then it requires a higher pilot power from each operator. Without loss of generality, we denote by $c^*$ the addition cost imposed by the Nash equilibrium compared to the standard pilot power setting $P^s$. The cost $c^*$ includes both the operating and the subjective costs. Recall that we defined the strategic gain $g_A$ in Section IV-A. Due to symmetry $g_A = g_B$, hence we denote it by $g$. In the extended game $\hat{G}$, we assume that the strategic gain is higher than the corresponding cost of using higher pilot power, thus $g > c^*$.

We present the payoff matrix of the game $\hat{G}$ in Table III. In each payoff pair, the first payoff belongs to player $A$, whereas the second to player $B$.

TABLE III
Payoff matrix of the game $\hat{G}$.

| | | Player B | |
|---|---|---|---|
| | | $P^s$ | $P_2^*$ |
| Player A | $P^s$ | $\Pi$,$\Pi$ | $\Pi - g$,$\Pi + g - c^*$ |
| | $P_1^*$ | $\Pi + g - c^*$,$\Pi - g$ | $\Pi - c^*$,$\Pi - c^*$ |

To emphasize the structure of the payoff matrix, let us substitute the values $\Pi = 3$, $g = 2$ and $c^* = 1$. Hence, we obtain the numerical values shown in Table IV. From the payoff matrix, one can realize that the game $\hat{G}$ is equivalent to the well-known Prisoner's Dilemma [8], [9], [25]. Analogously, the strategy $P^s$ corresponds to cooperation, whereas the strategy $P_i^*$ corresponds to defection. Basically, this also means that all the techniques that are used to resolve the classic Prisoner's Dilemma problem apply to the extended power control game $\hat{G}$ as well.

TABLE IV
THE EXTENDED POWER CONTROL GAME $\hat{G}$ CORRESPONDS TO THE PRISONER'S DILEMMA.

| | | Player B | |
|---|---|---|---|
| | | $P^s$ | $P_2^*$ |
| Player A | $P^s$ | 3,3 | 1,4 |
| | $P_1^*$ | 4,1 | 2,2 |

## VII. CONCLUSION

In this paper, we have have studied the of pilot power control of operators of CDMA networks. In particular, we have focused on the potential conflict of two operators that operate their networks on the two sides of a national border. We have investigated the question if the two operators should adjust their pilot signal powers (i.e., to be strategic) or not. To get an insight into the problem, we have considered the single cell-case with two base stations. Initially, we assumed that only one operator can adjust the pilot signal power of his base station. We have shown that he should be strategic and quantified the effect of various parameters on his strategic gain. We have shown that if both operators are strategic and the user density is low, then being strategic or not (i.e., the non-cooperative Nash equilibrium and the standard pilot powers) result in similar utilities. We have recognized that the two solutions require different pilot powers. If the user density is high, then the Nash equilibrium is more efficient than the standard solutions which suggests that the operators should be strategic. Finally, we have extended the utility function to include the cost of using high pilot powers. We have establish the analogy between the power control game with power cost in case of low user densities and the well-known Prisoner's Dilemma.

In terms of future work, we will extend the study of the single cell case to scenarios including several base stations. Because this case is fairly complex, we will propose distributed algorithms to identify and achieve Nash equilibria. Furthermore, we will consider power control games for scenarios, where users are associated with one of the operators. Finally, we will study the enforcement of desirable power signal levels through power pricing.

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